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ABSTRACT

The Auto-Regressive Integrated Moving Average (ARIMA) Models, often referred to as Box-Jenkins models, are regression methods for analyzing sequential dependent observations with large amounts of data. The Box-Jenkins approach, a three-stage procedure consisting of identification, estimation and diagnosis, was used to select the most appropriate stochastic model for describing undergraduate grade point average. The findings, based on a half-century of data from two universities, showed that meaningful mathematical models can be created to describe the time series of changes in the yearly grade point average. The models are tentative because of the small number of available observations and their relative complexity. A mathematical description of the time series of grades is complex enough to suggest that no simple answer may suffice. The data appears to be best modeled by an approach that postulates random shocks persisting for only a finite time, yet each of which can be represented as an exponentially weighted average of all previous observation. (JAZ)

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TIME SERIES ARIMA MODELS OF UNDERGRADUATE GRADE POINT AVERAGE

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Abstract

The Box-Jenkins approach to time series analysis, a regression method for analyzing sequential dependent observations, was used to select the most appropriate stochastic model for describing undergraduate grade point averages. The technique, applied to approximately a half century of data from two universities, suggested that the moving average model provided the optimal fit. Suggestions were made for further exploration of GPA data.

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TIME SERIES ARIMA MODELS
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Whenever a phenomena is observed over time, it is often useful to search for temporal patterns within the data. Economists have studied stock market prices, sociologists have examined population levels, and psychologists have investigated changes in the incidence of depression. For such purposes, a variety of time series analysis procedures have been developed, derived primarily from the theory of multiple regression. These techniques require data gathered from at least fifty time periods (McCleary and Hay, 1980, p. 20). Since arcnival data covering this many time periods is not as commonly collected in education as in some other fields, these mathematical approaches are not as widely used in educational research. It is the purpose of this paper to illustrate such an application, using undergraduate grade point averages.

Although educational institutions evaluate their students each term, a single group of pupils is not often evaluated fifty times on the same variable, as would be required for a time series analysis. However, a meaningful time series can be realized by obtaining the average grades given during each of the grading periods across a lengthy time span. For about the last half century, many universities and colleges have adopted a 5-point grading scale, using either the letters A through E or the numbers 1 through 5. Some of the institutions calculated, at each grading period, the average of grades awarded to their students, with the intent of maintaining reasonable consistency in their grading standards both among their departments and across time. Approximately fifteen years ago, reports began appearing that a conspicious increase was occuring each year in the grading patterns at many institutions (Birnbaum, 1977). Although that pattern appears to have abated during the past few years (Suslow, 1977), grades remain at a noticably higher level than

prior to the increase.

A variety of factors have been suggested to explain the phenomena of institutional grade average fluctuation (Birnbaum, 1977), but there has been a lack of data that support the proposed explanations. Rogers (1983) examined several independent variables (demographic and economic) for the possibility of explaining temporal variation over an extended time frame, but found each of them lacking in explanatory power.

Any "explanation" of a phenomena implies that the phenomena can be adequately described. Mathematical models, and regression models in particular, are appropriate for such a description, but an examination of the literature suggests that most authors rely solely on visual graphs rather than employing mathematical modeling. It was the purpose of this study to use a stochastic time series approach to generate mathematical models that might appropriately describe the entire sequence of grade point data.

Method

Sample

Grade point average data were collected from two midwestern universities for about a fifty year span. For the first, hereafter called University A, data was collected for each year from 1929 through 1982. This data is plotted as a time series plot in Figure 1. For the second institution, hereater called University B, data was collected each year from 1932 to 1982, except for the years 1943 through 1946, when no data was available. This data is plotted in Figure 2.

Procedure

These data were analyzed with the time series analysis procedures brought together in 1970 by George E. P. Box and Gwilyn M. Jenkins, in their volume entitled Time Series Analysis: Forecasting and Control (revised

edition 1976). These Auto-Regressive Integrated Moving Average (ARIMA) models (often referred to as "Box-Jenkins" models) require a large amount of data. However, when data are collected over an extended time period, as in this study, there is the possibility that the social meaning of the data could change over time. Thus, it becomes difficult to assign the same interpretation to the data at the beginning and end of the series. Nonetheless, the study of temporal patterns is an intriguing one, and with the development of appropriate computer software, the Box-Jenkins methods have become available to a much wider audience.

McCleary and Hay (1980) have prepared a treatise designed to encourage the use of the Box-Jenkins analysis for social science data, and to explicate strategies for both analyzing the data on the computer and presenting the computer output. Their strategies undergird the analysis in this study. The data was processed on a Harris computer, using MINITAB (Ryan, et al., 1982). Other approaches and other computer programs could have been used, but this was the one available for this project. The reader will need to interpret the methodological procedure of this study in that light.

The empirical identification procedures recommended by Box and Jenkins require an analysis of the autocorrelation function (ACF) and the partial autocorrelation (PACF) of the time series. The graphed ACF and PACF for both of the University time series are shown in Figures 3 and 4. The ACF is a set of correlations, each one of which represents the correlation between the original sequence and itself when lagged
k units. For observations close together, e.g., 1 or 2 lags, we most often find a higher correlation than for observations further apart, as is typified in Figures 1 and 2, where the correlations are slowly dying out as the lags

increase. This dying out phenomena is a consequence of the fundamental tenent of the ARIMA model, namely that the effect of any given input to the system declines over time. (Note that this is just the opposite of a time series of a bank savings account where, assuming a constant interest rate, the compounded interest from the first dollar invested is always larger than that from any subsequent dollar invested.) When the data is properly modeled, the residuals (errors resulting from the model) should be randomly distributed, and thus yield an ACF with with values that are all statistically non-significant. The goal of the Box-Jenkins approach is to find such a model.

The Box-Jenkins approach is a three stage procedure to build a model, consisting of Identification, Estimation, and Diagnosis. Each of these will be illustrated in the following analysis. The cycle iterates until an interpretable solution is found.

University A

Identification.

An examination of the ACF of the raw data (Figure 3) shows that the ACF falls to zero slowly, indicating that there is a strong systematic trend in the data. The most common method for removing this trend is to transform the data by replacing each observation with the difference between it and the preceding observation. When this differencing transformation is complete, the ACF is again computed. Figure 5 shows the ACF for the differences. The values are much smaller, indicating almost random data. However, there are some spikes, which may be due to sampling error or to some systematic process, so further analysis is required.

The PACF is interpreted similar to the ACF, except that each value is the correlation between observations k units apart after the correlation at

intermediate lags has been controlled or "partialled out". The PACF in Figure 3 shows a single spike, which may be the result of what is called a moving average (MA) component. This moving average component can be conceptualized as a random "shock" which is added to each observation to obtain the predicted value for the next observation.

The distinguishing characteristic of a moving average process is the finite duration of the shock. The shock persists for q observations and then is completely suppressed (McCleary and Hay, p. 61). Such a "shock" might be the result of the new grades that are added each term for each particular student. Since the majority of students will leave the institution after four years, the impact of any particular student will vanish when that individual leaves.

From the ACF and PACF we can now tentatively "identify" the model as an ARIMA (0, 1, 1). The zero indicates that there is no auto regressive (AR) term, the middle 1 indicates that differencing is to be used (this is the Integrative (I) term), and the last 1 indicates a moving average (MA) term.

Estimation.

When the estimates of the parameters were computed, it was found that the (0, 1, 1) model produced a t-value of only 1.23 for the MA term. Since this value was not statistically significant at the .05 level (nor anywhere near there), the model was rejected, and the procedure returned to the identification stage.

Identification.

It might be useful at this point to emphasize that since the estimated ACF and PACF are based on very small samples, they are subject to relatively large sampling errors. Consequently, any identification is very tentative.

Because the ACF and PACF for first differences appeared rough, it seemed appropriate to take second differences, i.e., differences between the difference scores. Figure 6 shows the resulting ACF and PACF. They appear more interpretable, suggesting a (0, 2, 1) model. An examination of Figure 1 also suggested that the variance was not constant across time. To attempt to correct this, a logarithmic transformation of the data was performed.

Estimation.

Table 1 shows the results of estimating the (0, 2, 1) model. The moving average parameter of .9767 satisfies the stationarity requirement that its absolute value be less than 1.0, and is also statistically significant at less than the .05 level.

Diagnosis.

The simplest diagnostic procedure is to compare the results of the given model and alternative models. In this way, it can be shown that a particular model is optimal in that neither a simpler nor a more complex model will suffice. The simpler model (0, 1, 1) was already shown to be inadequate. The more complex model (0, 2, 2) yielded a statistically insignificant second MA term, so it was rejected. The (1, 2, 1) model was also tested, but the AR term was insignificant. Thus, the ARIMA (0, 2, 1) model was accepted as the "best" fit.

The equation generated by this procedure can be conveniently written in the following form: $(1-B)^2 y_t = (1-.9767B)a_t$ where B is the backshift operator, and a_t is the random-shock element (McCleary and Hay, (1980), p. 46, 64). The backshift operator is defined as $By_t = y_{t-1}$ and follows the usual algebraic rules. The operator (1-B) represents first differences and $(1-B)^2$ represents second differences.

The random shock element a_t is the stochastic component in the equation. In the ARIMA model this moving average component can be shown to be mathematically equivalent to the exponentially weighted average of all previous observations (Pankratz, 1983, p. 49, 109; McCleary and Hay, (1980), p. 63).

University B

Identification.

An examination of the estimated ACF and PACF of the raw data (Figure 4) suggests that this data is also non-stationary and needs to be differenced. The single spike on the PACF suggests a (0, 1, 1) model.

Estimation.

The (0, 1, 1) model produced an estimate of the Moving Average parameter with a t-value of .23. Since this was far from statistical significance, modifications needed to be made. Second differences were used, since the data appeared to approximate a quadratic trend. The (0, 2, 1) model produced a parameter with a t-value of 11.12, which was highly significant.

Diagnosis.

The model was first diagnosed by comparing it with a more complex model. Accordingly, a (0, 2, 2) model was tested. It produced significant t-values for both MA parameters, as shown in Table 1. To compare the two models, the mean squares of the residuals was computed. The (0, 2, 1) model yielded MSR = .0011274, while the (0, 2, 2) model yielded MSR = .0009977. Finally, a (1, 2, 2) model (yet more complex) was tested, but it yielded MSR = .0011641. Consequently, the (0, 2, 2) model was favored, since it yielded the smallest MSR.

The ACF and PACF for the Residuals of model (0, 2, 2) are shown in Figure 7. No spikes are shown at lag 1 or any other lags. The residuals appear to meet the diagnostic criteria, so the model is accepted.

The model can be conveniently written as $(1-B)^2 y_t = (1 - 1.1475B + .5302B^2)a_t$.

Seasonality

The data analyzed above were yearly data, but many social science data are collected on a monthly or quarterly basis and show strong seasonal components. To test the hypothesis that a seasonal trend occured in grade averages within each year, data was obtained from University B for each quarter term for a 44 year span. The raw data plot is shown in Figure 8, where the numbers 1, 2, and 3 designate Fall, Winter, and Spring terms, respectively. A strong seasonal trend appears to be a noticable feature, so a time series seasonality analysis was performed. Seasonality is defined as "any cyclical or periodic fluctuation in a time series or repeats itself at the same phase of the cycle or period" (McCleary and Hay, 1980, p. 80).

Identification

Figure 9, showing the estimated ACF and PACF of the first differenced data, indicates a strong seasonal trend. The seasonality factor is known to be three. Further, most social science process yield regular and seasonal factors of the same type. Since the previous analysis of University B suggested ARIMA (0, 2, 2) model, it was decided to first test the ARIMA $(0, 2, 2)\ (0, 2, 2)_3$ model. However, that model produced an estimate of a Moving Average parameter with a t-value of only .41. Modifications were then made upon the model. After several trials, the ARIMA $(0, 1, 1),(0, 1, 1)_3$ model was found to produce statistically significant terms, as shown in Table 3, with MSR = .0006550.

Diagnosis

The $(0, 1, 1),(0, 1, 1)_3$ model was diagnosed by comparing it with several competitive models. Adding a second MA parameter proved non-significant.

Using an AR term in either the regular or the seasonal component (which might be inferred from the slow decay in the ACF) yielded a significant term, but the MSR term was slightly larger. A simpler model, postulating only a seasonal term (0, 1, 0), $(0, 1, 1)_3$ also produced a larger MSR.

The ACF and PACF for the residuals of the model are shown in Figure 10. Since no spikes are observed, the model is accepted.

A time series with both regular and seasonal components is expressed as a multiplicative model. The accepted model is written as:

$(1-B)(1-B^3)y_t = (1-.37B)(1-.77B^3)a_t$.

Conclusion

This paper has suggested that meaningful mathematical models can be created to describe the time series of changes in the yearly grade point average at a university. The models are very tentative, partly because of the small number of available observations and also because of their relative complexity.

While this paper has not answered the questions about the so-called "grade inflation," it has indicated that a mathematical description of the time series of grades is sufficiently complex to suggest that no simple answer may suffice. The data is unstationary, as shown by the need for differencing. It further appears to be best modeled by an approach that postulates random shocks that persist for only a finite time, yet each of which can be represented as an exponentially weighted average of all previous observations. This perhaps reflects both the influx of new students and the persistent effects of traditional grading practices.

Data for this study was available for only two institutions of higher education, so the generalizability of the results is limited. Studies with data from other institutions would serve to indicate the existence of general patterns across institutions.

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Table 1. Parameter estimates for the ARIMA (0, 2, 1) model. University A.

```
FINAL ESTIMATES OF PARAMETERS
NUMBER      TYPE     ESTIMATE      ST. DEV.   T-RATIO
     1     MA  1       0.9767        0.0439     22.26

DIFFERENCING. 2 REGULAR

RESIDUALS.      SS =     0.0191286  (BACKFORECASTS EXCLUDED)
                DF =    51  MS =    0.0003751
NO. OF OBS.     ORIGINAL SERIES    54     AFTER DIFFERENCING    52
```

Table 2. Parameter estimates for the ARIMA (0, 2, 2) model. University B.

```
FINAL ESTIMATES OF PARAMETERS
NUMBER      TYPE     ESTIMATE      ST. DEV.   T-RATIO
     1     MA  1       1.1475        0.1224      9.38
     2     MA  2      -0.5302        0.1220     -4.35

DIFFERENCING. 2 REGULAR
RESIDUALS.      SS =     0.0429018  (BACKFORECASTS EXCLUDED)
                DF =    43  MS =    0.0009977
NO. OF OBS.     ORIGINAL SERIES    47     AFTER DIFFERENCING    45
```

Table 3. Parameter estimates for the ARIMA (0, 1, 1), $(0, 1, 1)_3$ model University B.

```
FINAL ESTIMATES OF PARAMETERS
NUMBER      TYPE     ESTIMATE      ST. DEV.   T-RATIO
     1     MA  1       0.3704        0.0814      4.55
     2    SMA  3       0.7728        0.0564     13.70

DIFFERENCING. 1 REGULAR    1 SEASONAL DIFFERENCES OF ORDER  3

RESIDUALS.      SS =     0.0825284  (BACKFORECASTS EXCLUDED)
                DF =   126  MS =    0.0006550
NO. OF OBS.     ORIGINAL SERIES   132     AFTER DIFFERENCING   129
```

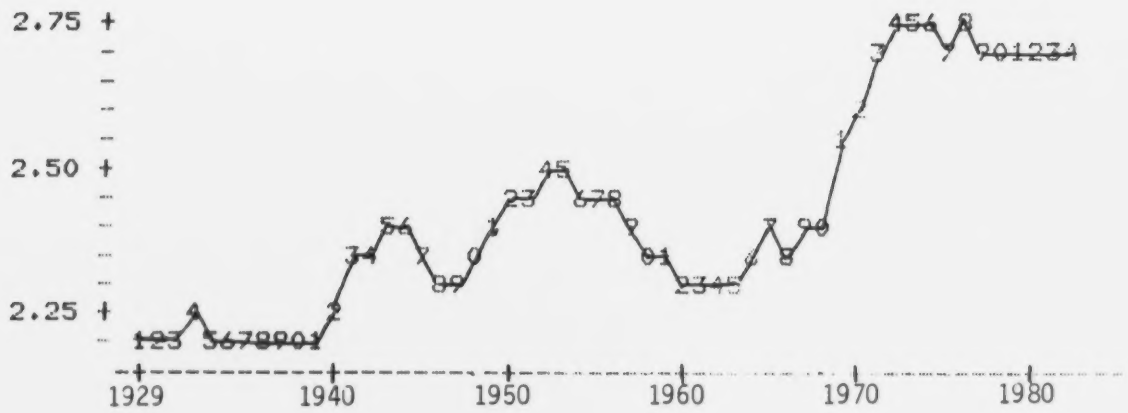

Figure 1. Grade Point Average (GPA) at University A, by year, from 1929 to 1982. (Prior to 1944 the data is for the whole year; afterward it is for fall term.)

Figure 2. GPA at University B, by year, from 1932 to 1982 (fall term). Far 1943-1946, data are not available.

Autocorrelations

```
                -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
                  +----+----+----+----+----+----+----+----+----+----+
 1    0.931                               XXXXXXXXXXXXXXXXXXXXXXX
 2    0.849                               XXXXXXXXXXXXXXXXXXXXX
 3    0.753                               XXXXXXXXXXXXXXXXXXX
 4    0.666                               XXXXXXXXXXXXXXXXX
 5    0.567                               XXXXXXXXXXXXXXX
 6    0.484                               XXXXXXXXXXXXX
 7    0.392                               XXXXXXXXXXX
 8    0.312                               XXXXXXXXX
 9    0.221                               XXXXXXX
10    0.119                               XXXX
11    0.026                               XX
12   -0.042                              XX
13   -0.084                             XXX
14   -0.112                            XXXX
15   -0.101                            XXXX
16   -0.083                             XXX
17   -0.060                             XXX
```

Partial Autocorrelations

```
                -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
                  +----+----+----+----+----+----+----+----+----+----+
 1    0.931                               XXXXXXXXXXXXXXXXXXXXXXX
 2   -0.140                            XXXXX
 3   -0.135                             XXXX
 4    0.028                                XX
 5   -0.145                            XXXXX
 6    0.064                                XXX
 7   -0.137                             XXXX
 8    0.017                                X
 9   -0.134                             XXXX
10   -0.185                           XXXXXX
11    0.051                                XX
12    0.066                               XXX
13    0.126                               XXX
14   -0.008                                X
15    0.237                               XXXXXX
16   -0.010                                X
17   -0.034                              XX
```

Figure 3. Estimated ACF and PACF for GPA. University A.

Autocorrelations

		-1.0 -0.8 -0.6 -0.4 -0.2 0.0 0.2 0.4 0.6 0.8 1.0
		+----+----+----+----+----+----+----+----+----+----+
1	0.939	XXXXXXXXXXXXXXXXXXXXX
2	0.899	XXXXXXXXXXXXXXXXXXX
3	0.827	XXXXXXXXXXXXXXXXX
4	0.749	XXXXXXXXXXXXXXXX
5	0.673	XXXXXXXXXXXXXX
6	0.595	XXXXXXXXXXXXX
7	0.511	XXXXXXXXXXX
8	0.436	XXXXXXXXXX
9	0.356	XXXXXXXX
10	0.291	XXXXXXX
11	0.220	XXXXX
12	0.174	XXXX
13	0.129	XXX
14	0.093	XX
15	0.060	XX
16	0.032	X

Partial Autocorrelations

		-1.0 -0.8 -0.6 -0.4 -0.2 0.0 0.2 0.4 0.6 0.8 1.0
		+----+----+----+----+----+----+----+----+----+----+
1	0.939	XXXXXXXXXXXXXXXXXXXXX
2	0.142	XXX
3	-0.263	XXXXXXX
4	-0.180	XXXX
5	0.003	X
6	0.000	X
7	-0.113	XXX
8	-0.002	X
9	-0.042	X
10	0.043	X
11	-0.069	XX
12	0.116	XXX
13	0.039	X
14	-0.040	X
15	-0.060	X
16	-0.006	X

Figure 4. Estimated ACF and PACF for GPA. University B.

Autocorrelations

```
                -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
                 +----+----+----+----+----+----+----+----+----+----+----+
 1    0.128                                XXXX
 2    0.141                                XXXXX
 3    0.046                                XX
 4    0.029                                XX
 5   -0.322                          XXXXXXXXX
 6   -0.008                                X
 7   -0.111                             XXXX
 8   -0.065                              XXX
 9    0.086                                XXX
10   -0.166                             XXXXX
11   -0.157                             XXXXX
12    0.008                                X
13   -0.104                              XXXX
14   -0.265                           XXXXXXXX
15    0.025                                XX
16   -0.085                              XXX
17   -0.035                               XX
```

Partial Autocorrelations

```
                -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
                 +----+----+----+----+----+----+----+----+----+----+----+
 1    0.128                                XXXX
 2    0.126                                XXXX
 3    0.015                                X
 4    0.004                                X
 5   -0.345                          XXXXXXXXXX
 6    0.071                                XXX
 7   -0.038                               XX
 8   -0.025                               XX
 9    0.158                                XXXXX
10   -0.354                          XXXXXXXXXX
11   -0.100                             XXXX
12    0.068                                XXX
13   -0.145                             XXXXX
14   -0.133                             XXXXX
15   -0.081                              XXX
16   -0.162                             XXXXX
17    0.043                                XX
```

Figure 5. Estimated ACF and PACF for first differences. University A.

Autocorrelations

```
                -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
                  +----+----+----+----+----+----+----+----+----+----+
 1   -0.505              XXXXXXXXXXXXXX
 2    0.073                        XXX
 3   -0.063                      XXX
 4    0.196                        XXXXXX
 5   -0.387                XXXXXXXXXXX
 6    0.251                        XXXXXXX
 7   -0.097                      XXX
 8   -0.057                       XX
 9    0.237                       XXXXXXX
10   -0.139                     XXXX
11   -0.085                      XXX
12    0.143                        XXXXX
13    0.037                        XX
14   -0.265                   XXXXXXXX
15    0.217                        XXXXXX
16   -0.097                     XXX
17    0.036                       XX
```

Partial Autocorrelations

```
                -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
                  +----+----+----+----+----+----+----+----+----+----+
 1   -0.505              XXXXXXXXXXXXXX
 2   -0.244                    XXXXXXX
 3   -0.201                     XXXXXX
 4    0.118                         XXXX
 5   -0.318                   XXXXXXXXX
 6   -0.139                      XXXX
 7   -0.131                      XXXX
 8   -0.287                    XXXXXXXX
 9    0.236                         XXXXXXX
10   -0.073                       XXX
11   -0.179                     XXXXX
12    0.039                         XX
13   -0.018                         X
14   -0.072                       XXX
15   -0.028                        XX
16   -0.206                    XXXXXX
17    0.021                         XX
```

Figure 6. Estimated ACF and PACF for second differences. University A.

Autocorrelations

```
                -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
                 +----+----+----+----+----+----+----+----+----+----+----+
 1    0.157                                XXXXX
 2   -0.056                               XX
 3   -0.098                              XXX
 4   -0.023                               XX
 5   -0.122                             XXXX
 6   -0.094                              XXX
 7   -0.131                             XXXX
 8   -0.065                              XXX
 9   -0.055                               XX
10   -0.094                              XXX
11   -0.012                                X
12    0.125                                XXXX
13    0.152                                XXXXX
14    0.064                                XXX
15   -0.086                              XXX
16   -0.013                                X
```

Partial Autocorrelations

```
                -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
                 +----+----+----+----+----+----+----+----+----+----+----+
 1    0.157                                XXXXX
 2   -0.082                              XXX
 3   -0.078                              XXX
 4    0.001                                X
 5   -0.135                             XXXX
 6   -0.065                              XXX
 7   -0.131                             XXXX
 8   -0.065                              XXX
 9   -0.079                              XXX
10   -0.142                            XXXXX
11   -0.034                               XX
12    0.059                                XX
13    0.065                                XXX
14    0.003                                X
15   -0.123                             XXXX
16    0.002                                X
```

Figure 7. Estimated ACF and PACF for residuals from Arima (0, 2, 2) model. University B.

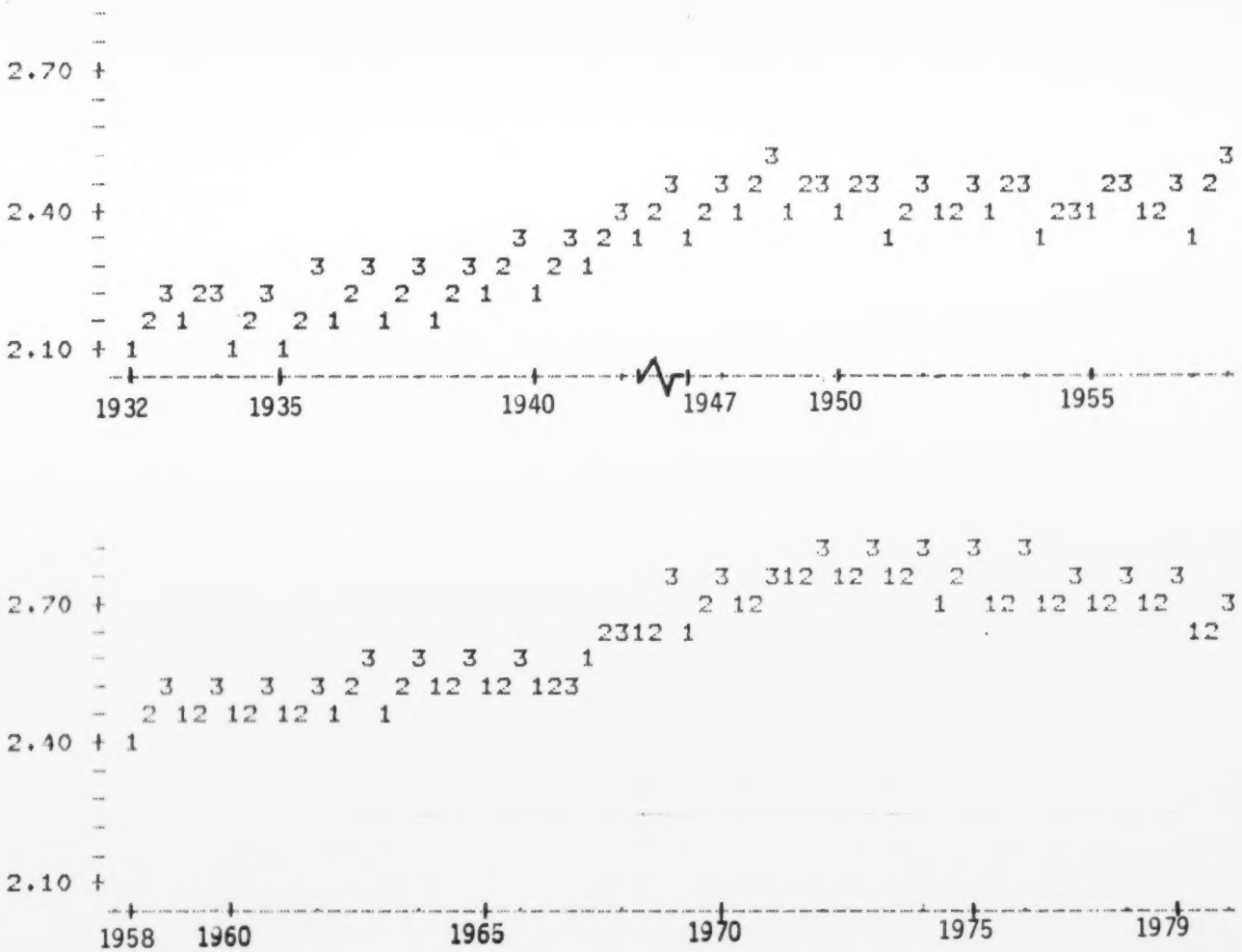

Figure 8. GPA at University B, by quarter, from fall term 1932 to spring term 1982. For 1943-1946, data are not available.

Autocorrelations

```
                -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
                  +----+----+----+----+----+----+----+----+----+----+
 1   -0.461                XXXXXXXXXXXX
 2   -0.426                 XXXXXXXXXXX
 3    0.857                           XXXXXXXXXXXXXXXXXXXX
 4   -0.419                 XXXXXXXXXX
 5   -0.427                 XXXXXXXXXXX
 6    0.808                           XXXXXXXXXXXXXXXXXXX
 7   -0.375                  XXXXXXXXX
 8   -0.396                 XXXXXXXXXX
 9    0.762                           XXXXXXXXXXXXXXXXXX
10   -0.351                  XXXXXXXXX
11   -0.383                 XXXXXXXXXX
12    0.729                           XXXXXXXXXXXXXXXXX
13   -0.351                  XXXXXXXXX
14   -0.374                  XXXXXXXXX
15    0.707                           XXXXXXXXXXXXXXXXX
16   -0.344                  XXXXXXXXX
17   -0.343                  XXXXXXXXX
18    0.644                           XXXXXXXXXXXXXXXX
19   -0.291                    XXXXXXX
20   -0.360                  XXXXXXXXX
```

Partial Autocorrelations

```
                -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
                  +----+----+----+----+----+----+----+----+----+----+
 1   -0.461                XXXXXXXXXXXX
 2   -0.811           XXXXXXXXXXXXXXXXXXXX
 3    0.471                           XXXXXXXXXXXX
 4    0.044                           XX
 5   -0.256                      XXXXXXX
 6    0.061                           XXX
 7    0.037                           XX
 8    0.043                           XX
 9    0.102                           XXXX
10    0.020                           X
11   -0.007                           X
12    0.108                           XXXX
13   -0.037                         XX
14   -0.090                        XXX
15    0.023                          XX
16   -0.039                         XX
17    0.029                          XX
18   -0.175                       XXXXX
19    0.042                          XX
20   -0.115                        XXXX
```

Figure 9. Estimated ACF and PACF for first differences on quarter data. University B.

Autocorrelations

		-1.0 -0.8 -0.6 -0.4 -0.2 0.0 0.2 0.4 0.6 0.8 1.0
1	-0.021	XX
2	0.008	X
3	0.179	XXXXX
4	0.029	XX
5	-0.112	XXXX
6	-0.015	X
7	0.083	XXX
8	0.018	X
9	-0.037	XX
10	0.091	XXX
11	0.077	XXX
12	-0.020	X
13	-0.082	XXX
14	0.001	X
15	0.032	XX
16	-0.126	XXXX
17	0.054	XX
18	-0.144	XXXXX
19	0.052	XX
20	-0.125	XXXX

Partial Autocorrelations

		-1.0 -0.8 -0.6 -0.4 -0.2 0.0 0.2 0.4 0.6 0.8 1.0
1	-0.021	XX
2	0.008	X
3	0.179	XXXXX
4	0.038	XX
5	-0.118	XXXX
6	-0.056	XX
7	0.078	XXX
8	0.069	XXX
9	-0.022	XX
10	0.046	XX
11	0.059	XX
12	0.007	X
13	-0.105	XXXX
14	-0.047	XX
15	0.053	XX
16	-0.067	XXX
17	0.053	XX
18	-0.202	XXXXXX
19	0.077	XXX
20	-0.121	XXXX

Figure 10. Estimated ACF and PACF for residuals from the ARIMA model $(0, 1, 1), (0, 1, 1)_3$. University B.